The Effects of Using Dynabook to Prepare Special Education Teachers to Teach Proportional Reasoning

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ABSTRACT

The Dynabook research project was a collaborative effort aimed at developing and evaluating a new interactive Web-based resource for use in teacher education to improve how teachers learn to teach middle-school mathematics. This paper describes the outcomes of research during a three-year project to develop and evaluate the effectiveness of the "Dynabook." The purpose of this mixed methods study was to understand how special education college faculty used the proportional Dynabook and how it affected the learning of teacher candidates in their courses, as it was being designed and modified over a period of three years. In the results, faculty note an improvement in the credential candidates' skills in solving proportional problems and answering pedagogical content knowledge questions, and in their mathematical thinking as measured in complexity of classroom discussions. The candidates also reported feeling more confident in their teaching of proportional reasoning, their implementation of UDL, and understanding of TPACK.

Keywords: *Dynabook, Dynabook Research Project, Mathematics, Special Education, Teacher Education*

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USING THE PROPORTIONAL DYNABOOK TO HELP SPECIAL EDUCATION TEACHERS LEARN TO TEACH MIDDLE SCHOOL MATHEMATICS

The Dynabook Project is a collaborative research effort aimed at understanding a new interactive Web-based resource for use in teacher education to improve how teachers learn to teach middle school mathematics. The full team included, a) faculty from two California state universities (CSU's); b) researchers, mathematicians, and technical designers from SRI International, a nonprofit organization that has created digital materials for learning mathematics using dynamic graphical representations (Roschelle, 2010; Roschelle, Shechtman, & Tatar, et al., 2010); c) CAST, another nonprofit organization that has developed digital materials, including a framework for creating these materials using Universal Design for Learning UDL (Rose & Meyer, 2002; Rose, Meyer, & Hitchcock, 2005); and d) Inverness Research, the external evaluator.

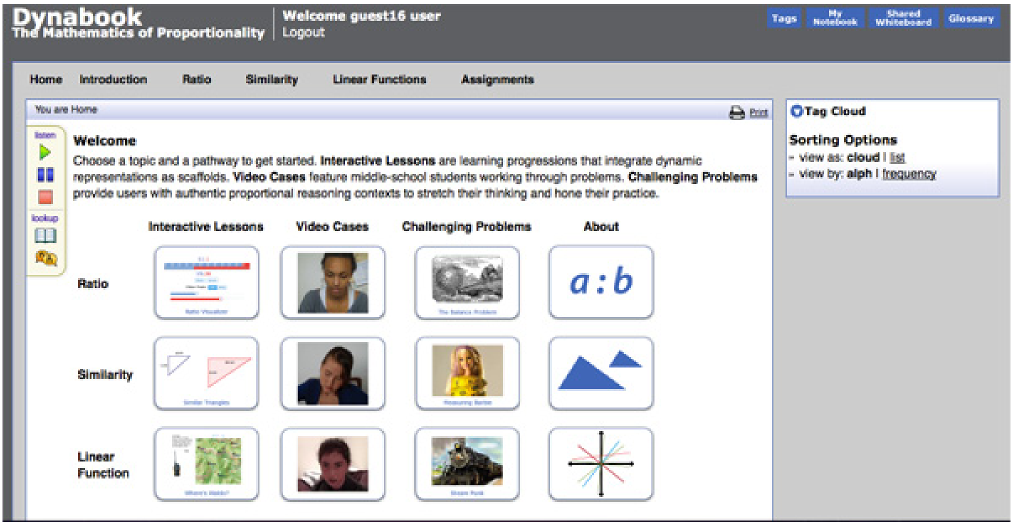
This article presents the research collected from the special education (SPED) students (pre-service special education teachers). The SPED faculty members took part in this three-year grant project developing and evaluating an electronic math textbook called the "Dynabook." The purpose of this paper was to examine what the SPED faculty learned over the three years while they were assisting with designing and examining the effects of this mathematics tool with their graduate credential candidate students.

The original goal of developing a digital textbook changed quickly when the team first came together five years ago. First, it became clear that a digital textbook (traditionally conceived) would not take advantage of the multimedia capabilities of the Web in the way that was most effective. Second, there was confusion on whether the technology should be designed for use by the college faculty, the K-12 teachers, or both. It wasn't until individual team members came together that they realized the needs of those two groups overlapped in some ways, but were quite different in others, so they chose to target teacher educators. Third, the teacher educators on the team, two math professors who taught liberal studies undergraduates and two special education professors who taught graduate students working on a credential to teach students with learning differences, had varied challenges. Both groups of faculty members were preparing students to become teachers but had very different needs and goals. The math professors were preparing undergraduates who might become middle school math teachers in general education classrooms. The special education faculty were preparing pre-service and in-service teachers who teach math as well as all other content areas for students with learning differences in kindergarten through grade 12. The new direction was to develop something that worked for both groups.

The current Dynabook includes dynamic representations (for example an interactive ratio bar), videos of children solving problems, a shared workspace (like a built in white board), examples of practice problems and something the developers call "Dynalogue," a feature that allows students to animate two characters so they can engage in problem solving scenarios. These features were intended to help teacher educators instruct middle school teachers about how children think about proportionality, including their misconceptions and the strategies they used when solving problems. In addition, university instructors could create a social learning environment for teacher candidates so they could work together and present their ideas openly through this shared-work space.

The revised interface of the "book" starts with a 3x4 graphic matrix with different pathways where users can more efficiently enter the resource and access areas of particular interests. The rows of the matrix are set up in three strands of middle school mathematics that develop students' proportional thinking: ratio (in the number strand), similarity (in the geometry strand), and linear function (in the algebra strand). The columns provide four pathways through the content: 1) challenging

Figure 1. Dynabook interface as of fall, 2013



problems, which are math problems of increasing difficulty that are designed for candidates to develop their own mathematical thinking; 2) video cases of middle school students solving typical ratio, similarity and linear function problems, showcasing a variety of strategies and misconceptions; 3) interactive lessons that are specially designed to take advantage of the dynamic medium of the Dynabook by presenting mathematical ideas in a visual and interactive format; and 4) a knowledge bank with definitions, commonly used representations and appropriate example contexts for the different topics; and 5) the Dynalogue feature which was added later that allows students to animate two characters and have them discuss a math problem.

LITERATURE

Pre-service Teachers' Thinking about Proportional Reasoning

When studying pre-service teacher's attitudes towards mathematical thinking as it relates to proportionality, researchers concluded that teacher candidates' need more preparation in mathematics instruction (Berk & Taber, 2009; Hines & McMahon, 2005; Watson, 2002; Greer & Meyen, 2009). In 2005, Hines and McMahon asked pre-service teacher candidates to look at children's work to assess mathematical thinking with regard to proportional reasoning and they found that teacher candidates regarded the consistent application of a factor or symbolic procedure as indication of higher developmental level even with students who used lower level computations. Only one teacher candidate questioned whether the middle school children understood the procedures they used. Some teacher candidates tended to rate children in the category of lower development when they used proportional reasoning solution strategies that the candidates did not understand or misinterpreted as incorrect. However, these results were mixed because some teacher candidates identified clearer explanations as higher development regardless of the explanation content. Middle school students who gave more than one solution method were also judged as developmentally more advanced.

Berk and Taber (2009) also worked to improve elementary teacher's flexibility with proportional reasoning. They defined flexibility as the ability to employ multiple solution methods across a set of problems, solve the same problem using multiple methods, and choose strategically from among methods so as to reduce computational demands. They planned to improve flexibility by varying the source of the solution methods and then allowing teacher candidates to compare solution methods. They developed two versions of an intervention that varied only in the source of the solutions. In the Generated Solutions (GS) condition, prospective teachers generated and compared their own solutions to contextual missing-value proportion problems. In the Worked Solutions (WS) condition, prospective teachers compared worked solutions presented to them as middle school students' solutions to the same problems. The teacher candidates were interviewed to determine which solution methods they perceived to be better and why. Ultimately, the intervention helped candidates to improve in all categories of flexibility in mathematical thinking.

There are many ways to measure how students think about mathematics (Watson, 2002; Groth & Bergner, 2006; Kazemi & Stipek (2008/2009). Watson (2002) used multiple means of measurement in his study looking at children's mathematical thinking. One part of the measurement included habits of mind with items asking the student whether he or she believed that practicing with multiple types of proportional problems was important. Answering affirmatively was considered a sign of more sophisticated mathematical thinking.

When looking at teacher candidates' knowledge and understanding of basic statistics, Groth and Bergner (2006) developed a taxonomy that included four levels of sophistication. The most naïve level was called unstructural/concrete and symbolic and the responses relied solely on discussing the process for each measure. The second level was multistructural/concrete and symbolic with responses that included process-telling along with a vague notion that mean, median, and mode can all be used as data analysis tools. The third level was relational/concrete and symbolic and these responses included process-telling along with the idea that mean, median, and mode all measure the center of a data set and/or what is typical about the set. Finally, the highest level was called extended abstract and responses went beyond explaining the process to discussing situations when one of the three measures --mean, median, mode--might be a better measure of center and/or typicality than another. Only a few teacher candidates exhibited more abstract thinking than that of typical elementary school students by identifying hypothetical situations in which one would be a more appropriate measure of center than another.

Kazemi and Stipek (2008/2009) conducted a qualitative research study using videotapes of lessons for fractions. In their study, the differences between a more significant push and less significant push (student teacher interaction) when pressing for conceptual thinking was characterized by the following norms; (a) explanations consisted of a mathematical argument, not simply a procedural description or summary; (b) mathematical thinking involved understanding relations among multiple strategies; (c) errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, or pursue alternative strategies; and (d) collaborative work involved individual accountability and reaching consensus through mathematical argumentation.

Researchers define and measure mathematical thinking in many different ways and it often depends on context. Consequently, getting a precise measure of teachers' and students' levels of mathematical reasoning is difficult and apparently dependent on the area of math and its theoretical lens.

Special Education and Middle School Mathematics

Teaching middle school mathematics to students eligible for special education services can be an even more formidable task. Boyd & Bargerhuff (2009) completed an extensive literature review exploring research that intersects middle school mathematics and special education. They suggest that special education college faculty often instruct teacher candidates to teach children to solve problems procedurally, while in mathematics education, college faculty are working with teacher candidates in student-centered, constructivist ways to teach children to solve problems more conceptually. These researchers admit that learning procedures in mathematics is important, but that special education teachers tend to focus on procedures too much, in part because special education methodology is more likely to emphasize task analysis and specific, measurable objectives, often targeting procedural rather than conceptual skills. The propensity of special education to use these approaches is not surprising, given some of the common characteristics of students with disabilities, who often struggle in areas such as short-term memory, visual and auditory processing, and executive functions. Furthermore, past research in the field of special education has demonstrated more effective outcomes for students with disabilities when teacher-directed instruction is used (Kroesbergen & Van Luit, 2003).

Mathematics and Technology Tools

Teacher educators have been using technology to build online communities in college classrooms (Grossman & Arnold, 2011; LePage & Robinson, 2005; Sockett & LePage, 2002) and they have used technology for teaching (You-Jin & Honguk, 2010). In addition, math professors have been using technology as tools to teach math in college classrooms (Ashline & Frantz, 2009; Bowers & Nickerson, in press). Dynabook brings various technology components together in an easy to use and dynamic way.

In her article, Suh (2010) explains that technology can enhance mathematics learning. Quoting Roy Pea, she writes, "cognitive" technology tools have been described as "technologies that help transcend the limitation of the mind... in thinking, learning and problem solving activities" (Pea 1985, p. 168). She also references Zbiek (2007), when she affirms that tools respond to a user's commands and make mathematical actions more overtly apparent. Cognitive mathematical technology tools are not simply the traditional remedial drill-and-practice computer programs (Suh, 2010). Users of these tools have the capability to graph, model, compute, visualize, simulate, and manipulate, which "amplify" mathematical properties and concepts. These cognitive technology tools have the potential to help students if teachers know how to use them properly (Zahner, & Corter, 2010). Conversely, teachers who do not know how to use supplemental activities and materials can actually lower the quality of their mathematics instruction (Hill et al., 2009)

In the special education courses where teachers are learning proportionality, they are most likely not learning new material, but working to recall information they have learned in the past. Halverson, Wolfenstein, Williams, & Rockman (2010) examined the use of technology to create digital learning objects (e.g., short tutorials and virtual manipulatives) that could spark the recall of mathematics previously learned. Inservice and pre-service special education teachers who were preparing to take the PRAXIS exam used the learning objects to help them remember how to access and apply math knowledge they had once learned. While the learning sciences have focused on questions of learning new knowledge, the context of how adults recover information has received less theoretical attention. The authors believed that to remember content once learned involves uncovering areas of "conceptual breakdown." Their theory is that the phenomenon of conceptual breakdown is different for remembering knowledge than for learning new knowledge. Remembering math involves reassembling misplaced, broken or fragmented conceptual knowledge once learned in school. The design of learning objects allowed them to determine which aspects of PRAXIS-type questions highlighted conceptual breakdown, and led the research team to build learning objects that would help learners reassemble prior concepts to improve capacity to solve similar problems.

These learning objects were effective at helping teachers improve their scores on the PRAXIS; pre-service teachers improved their scores by 27% and inservice teachers improved by 13%. However, they duly note that the learning objects were designed to help teachers recall math content learned in the past and to simply provide just enough content to pass the PRAXIS. The researchers caution that helping teachers recall math procedures does not mean that the teachers have learned and mastered mathematics well enough to be teaching students with learning challenges. Each learning object focused on a narrow area of conceptual or procedural knowledge required to solve a particular problem. Halverson, et al. (2010) acknowledged that these objects were not designed to provide a substitute for a more comprehensive approach to math education that helps learners acquire mathematical ways of knowing or flexible problem-solving skills.

Like Halverson et al. (2010), our team is also designing a digital mathematical resource for pre-service and in-service teachers. However, rather than just sparking recall of math knowledge learned in the past, we seek to deeply engage our credential candidates in thinking, talking, and reasoning about proportionality.

METHODS

Design

This article presents results from a mixed methods study born out of the larger development and design project. Special education professors at a state university in California agreed to use a newly designed and developed technology tool over the course of three years in their classroom and collect outcome data on this tool in their classes. In addition, researchers from SRI also collected formative evaluation data in those classrooms. Students were pre-service and in-service students in a graduate school of education. Over half of the students were already teaching in their own classrooms. All of the students were older, most often between 25 and 35, however the age range could extend from 25 to 50. Some of the students were in the process of changing careers to become special education teachers.

Instruments

Online Survey

Teacher candidates completed a survey of demographic items about their years of teaching experience, student grade levels and the severity and types of their children's documented disabilities, and the types of settings in which these students received services. The survey had items about the candidates' mathematics coursework in high school and college and survey items also asked how prepared students felt to teach elementary, middle school, high school, and post-secondary mathematics. They also answered items measuring self-efficacy for teaching proportional reasoning.

Learning Mathematics for Teaching (LMT)

As a quantitative measure, we used the Learning Mathematics for Teaching test (LMT) which was developed out of the Mathematics from Knowledge Test (MKT). The MKT measure was developed by Hill, Schilling and Ball (2004) and was developed to measure math pedagogical content knowledge, which according to research has a positive affect on student achievement (Hill, Rowan, Loewenberg Ball, 2005). The MKT measure consists of an online bank of items covering a variety of topics in mathematics. The Dynabook was designed (for this project) to assist professors in discussion-based collaborative learning of middle school mathematics concepts, so items measuring PCK were drawn from the LMT since the LMT was designed specifically to measure teachers' knowledge of middle school mathematics pedagogy. (For more information on the MKT and LMT, see Hill, Schilling, & Loewenberg Ball, 2004). Participants were asked to interpret students' mathematical reasoning from examples of hypothetical student responses to classroom instruction. For example, items on the LMT might describe a proportional reasoning problem presented by a hypothetical teacher and how several students responded to the problem.

Participants decided if the students were using correct reasoning or incorrect reasoning. There were also items in which participants viewed a sample problem and subsequent examples of student thinking; they must decide if the ideas model or fail to model *proportional* thinking. Thirteen items were chosen, many of them with multiple parts that yielded a total score of 32 points. A pretest and posttest were presented. The same items were distributed to participants in paper form at two points, in class before experiencing Dynabook lessons and after the Dynabook lessons had ended.

Data Collection Procedures

Data were collected in several ways. As part of the research pre and post mathematical pedagogical content tests were given to students before and after they were asked to use the new Dynabook system and self efficacy tests were also administered to students before and after the instruction. Students were also asked to complete online surveys after they finished using Dynabook in their classes. In addition, classes were videotaped and researchers conducted a discourse analyses on the classroom conversations from those videotapes. These methods are discussed in more detail below.

Collecting Data

LMT

The LMT was given as a pretest and then a posttest at the beginning and end of the classes where Dynabook was being integrated into the curriculum. These two measures were administered to students in a university course where faculty members were teaching special education teacher candidates how to teach children reading and math. Half the class was taught at the university and half the class was taught in an afterschool program at a middle school where teacher candidates worked with children who were facing learning challenges in math and reading. The same college professor taught the course each time the data was collected. The LMT test was presented before the teacher candidates started working with Dynabook and again after the instruction was completed. Data was collected only when instruction was provided over a two-day period.

Survey

The survey was on-line on Survey Monkey and students were asked to complete the survey on survey monkey usually after class. The survey included both Likert style questions and short answer essay questions. The surveys were revised several times over the research period to give the research team specific information they needed about the students' experiences over the course of the three years. So, not all data from the survey from different courses could be combined.

Observations and Videos

During the special education class sessions when Dynabook was being presented, and during one afterschool program at a middle school, multiple researchers from both SRI and from the university observed and took notes while also video taping the classes. Videos were transcribed verbatim so that transcripts could be analyzed. Notes were also kept during in-person meetings as researchers discussed the results of observations and video conclusions. Every class session was videotaped and members of the Dynabook team took observation notes in situ. This class was held once a week for three hours. Video was collected from at least two sides of the room allowing both the students to be video taped and the teacher. At least two people were taking notes at any one time during the research, usually more.

Teaching and Curriculum Procedures

The team developed several versions of the Web-based Dynabook through an iterative process. They were testing and experimenting with the use of the Proportionality technology tool in various teaching activities at both universities. In the special education classes, the formative

Table 1. Lessons and data collection by class and semester

Semester	Lesson Activities/Assignments	Version
Spring 2010	Candidates were exposed to the Dynabook prototype. They were given a short presentation by the SRI staff and allowed to browse the prototype. They were interviewed in small groups.	Prototype
Summer 2010	Candidates worked in pairs to write a ratio lesson plan using Dynabook. They navigated the text in groups of two and used Dynabook and other resources on the web (e.g., Wikipedia) to write the lesson plan.	Prototype Text heavy
Fall 2010	The Dynabook lesson occurred during one 3-hour class. Candidates took a lengthy pre-survey that included ratio problems and questions about self-efficacy for teaching ratio. They discussed the ratio problems as a whole group and students came up to the dry erase board to draw their solutions. The class discussed these solutions and gave feedback about the functionality and usability of the Dynabook interface. Then, instructors showed videos of students solving the problems and the candidates discussed the videos as a whole class.	Dynabook Ver. 1.0 More graphics added to interface Added videos of students doing work
Spring 2011	Day 1: Candidates worked in pairs and watched videos and read information about the accessibility and functionality of the Dynabook. Then, the whole class watched videos together to learn about the stages of proportional reasoning understanding and shifts between different levels. They went back to the Dynabook and began a second assignment, completing the first task, solving a ratio problem. After they worked with the Dynabook, they were given time to talk in class about what they learned. Day 2: Candidates posted answers to the ratio problem posed in the class the week before to a shared-work space and this class session began with a whole-group discussion about various solutions. The whole class then watched the video about the stages of proportional reasoning and shifts between these stages again. Then, the class watched a video of a student solving the ratio problem that they had solved. This was followed by a whole class discussion about the student's reasoning and how it relates to the stages of proportional reasoning. The class then divided into teams of two to write scripts in which they addressed the student's misconceptions and helped her transition to the next stage of proportional understanding. They Xtranormal to animate these scripts.	Dynabook Ver. 2.0 Added shared space and used animation program (Xtranormal to produce teaching scripts)
Spring 2011	The class began with a scavenger hunt of the functionality and usability of the Dynabook. The candidates worked in teams of two or three to watch videos and read content in Dynabook. In these small groups, they then started a second assignment that contained a context-rich ratio problem about mixtures. They solved this problem and posted answers and explanations to the shared-work space. When most groups had finished their responses, the whole class engaged in a discussion of the various explanations of the problem. Then, candidates worked on practice ratio problems found in the "Challenging Problems" section of the Dynabook. The class ended with a 15-minute discussion and debrief about the functionality of the Dynabook that included ideas for improvement for the next iteration.	Dynabook Ver. 2.1 Improved upon shared space where students could easily and immediately post on the shared work space
Summer 2011	We introduced Dynabook with a purposeful assignment that navigated candidates through various sections of ratio, they learned about the shifts in development of understanding proportional reasoning, solved problems, watched student video at various levels of understanding, and composed scripts of teachers advancing the students' understanding. Candidates were also observed in middle school summer program for children with learning and behavioral challenges.	Dynabook Ver. 2.1

continued on following page

Table 1. Continued

Semester	Lesson Activities/Assignments	Version
Fall 2011	Candidates were asked to solve a ratio problem and then share the problem anonymously by posting it on a shared space that resembled a whiteboard. Throughout the class, they continued to pair up and solve problems.	Dynabook 2.1
Spring 2012	Day 1: The class worked in small groups to solve a ratio problem with several embedded questions and activities. The problem required them to decide which school schedule was "mathier," given different schedules with ratios of math classes to other classes. They were asked to use an interactive tool called the ratio bar visualizer and present their work by posting it on the shared-work space. They watched a short video of a student incorrectly solving a ratio problem and talked to their small groups about her thinking. They began to work on an assignment to write a script that addressed that student's misconceptions. Day 2: Candidates arrived at class having completed scripts to address a student's misconceptions and animated the scripts using Xtranormal. The whole group discussed the experience of coming up with the scripts and then watched some videos about the stages of understanding in proportional reasoning and the shifts between these stages. Small groups edited their videos and then the whole class watched each video and evaluated them according to a rubric.	Dynabook Ver. 2.2 Students used Interactive Graphic manipulative to solve problems
Fall 2013	Used Dynalog, a new feature (like Xtranormal) embedded in the software	Dynabook Ver. 2.3

evaluation, research and redesign process was ongoing for two years as researchers provided formative evaluation feedback. Some features of the class lessons are briefly stated below.

Data Analysis

Quantitative Data

Students were asked to volunteer to complete an assessment that measures teachers' mathematical knowledge for teaching, called the LMT, which stands for Learning Mathematics for Teaching. This assessment tests both mathematics knowledge and teachers' mathematical pedagogical content knowledge. The test might give a teacher a ratio problem and then ask her how a child might misinterpret this problem. The test is given to a student both before and after the Dynabook lessons. Data was collected over four semesters with a total of 43 complete data sets from teacher candidates. A t-test was run on the average scores from the LMT data. Averages were calculated for the survey data; ultimately 102 surveys were collected and analyzed. The surveys were administered after the credential candidates completed all their work in Dynabook.

Qualitative Data

Videos transcriptions and observation notes were analyzed with a process of inductive discourse analysis. We coded the video transcriptions, identifying significant patterns and constructing a framework for communicating the main themes of the data (Patton, 2001). Inductive analysis means that the patterns, themes, and categories emerged out of the data rather than being imposed on them prior to data collection and analysis. Then, a content analysis was conducted which includes the process of identifying, coding, and categorizing the primary patterns in the data. In the final step, the data was interpreted. Interpretation, by definition, goes beyond description. Interpretation means attaching significance to what was found offering explanations, drawing conclusions, making inferences, building linkages, attaching meaning, imposing order and dealing with

Table 2. Circle the number below for 1) produces same ratio, 2) produces different ratio, or 3) i'm not sure for each

	Produced the Same Ratio	Produces Different Ratios	I Am Not Sure
a) The ratio of two people's heights, measured in (1) feet, or (2) meters.	**1**	**2**	**3**
b) The noontime temperatures yesterday and today, measured in (1) Fahrenheit, or (2) Centigrade.	**1**	**2**	**3**
c) The speeds of two airplanes, measured in (1) feet per second, or (2) miles per hour.	**1**	**2**	**3**
d) The growths of two bank accounts, measured in (1) annual percentage increase, or (2) end-of-year balance minus beginning-of-year balance.	**1**	**2**	**3**

rival explanations. Examples from the most sophisticated discussions during each session were the ones chosen to be included as evidence in this paper. In that way, researchers could show a progression in each group's thinking, providing examples of the most sophisticated dialog during each class.

Rigor and Credibility

An observation can be affected by observer bias. To account for limitations, qualitative researchers are expected to be rigorous in the way they collect, code, and analyze data. Within the positivist paradigm, a study's rigor is judged through measures of reliability and validity. Lincoln and Guba (1985) offer four alternative terms for determining rigor of a qualitative study: credibility, confirmability, dependability, and transferability. Triangulation of methods through the use of multiple cases, multiple observers, and multiple sources of data, and multiple theories strengthen the credibility of the results of this study (Lincoln, 1985).

The team involved in conducting the research, included special education faculty and other researchers from SRI who discussed the results to assist in data analysis. This allowed more than one researcher to confirm the results. This research was being conducted under unique circumstances and in a unique context. The purpose was to better understand the uses and the outcomes of a new type of technology for use in pedagogy and technology instruction for pre-service teachers. Other researchers can determine the applicability of our findings to their specific situations and contexts. Dependability refers to the researcher's attempts to account for changing conditions in the subject chosen for study. Miles and Huberman (1994) argue that researchers have their own understanding, their own convictions, and their own conceptual orientations. LeCompte and Preissle (1993) developed two terms, *emic* and *etic*. *Emic* means that the researcher must make every effort to understand the subjective meanings placed on the situations by participants. The term *etic* is concerned with the researcher's meaning and construction of the situations. Dependability means that the researcher can separate the two.

RESULTS

The results of the research are described here through the lenses of the various types of data collection methods. Each method of data collection brought about its own, although interrelated, insights into the usefulness and/or the effectiveness of the technology tool, as well as the characteristics of the teacher candidates and their needs, interests and abilities with regard to middle school mathematics, specifically ratio, which is what we focused on in this particular study.

The results of the LMT Assessment instrument pre- and post-tests suggest that a teacher candidate's PCK can improve through use of Dynabook during instruction. An example of a question on the LMT might look like the following (see sample responses in Table 2):

Mr. Garrison's students were comparing different rectangles and decided to find the ratio of height to width. They wondered, though, if it would matter whether they measured the rectangles using inches or measured the rectangles using centimeters.

As the class discussed the issue, Mr. Garrison decided to give them other examples to consider. For each situation below, decide whether it is an example for which different ways of measuring produce the same ratio or a different ratio.

The teachers LMT scores, which are meant to test both mathematics and pedagogical content knowledge (PCK) improved while the teachers were using Dynabook in the classroom, but were still quite low overall, given that the 43 students who were tested had an average earned 10.67 points out of 43 total during the pretest and they earned an average of 11.84 points in the post-test. The students on average improved significantly ($p<0.004$) after being taught about proportional reasoning using the Dynabook over two three-hour class sessions.

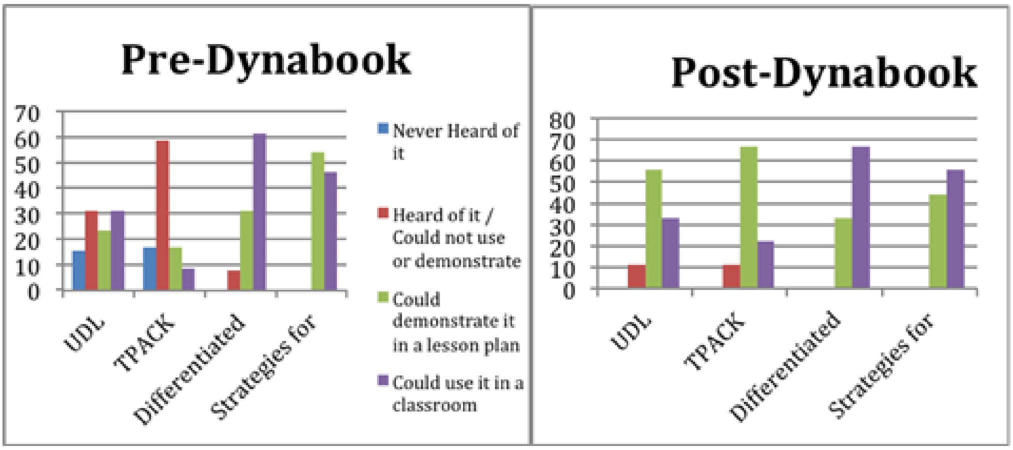
Survey Data

The surveys that were given to the students were often revised after each class, in order for the research team to collect the type of information they needed from different groups of students. The most interesting results of survey data collected in the Fall of 2012 was used to compare some demographic data from the students in the special education mild to moderate program Graduate College of Education (pre-service and inservice teachers) to those students' who were undergraduate from an undergraduate liberal studies program who planned to one day be a math teacher in middle school. In these two groups, students were very different. In the special education teacher preparation program, more than half the candidates were already teaching in their own classrooms; they were older students returning to school and their plans were to teach special education students in all content areas. The undergraduates were more traditional 18-22 year olds. These college students planned to go into a teacher education program after they completed their undergraduate degrees.

In this survey 46 undergraduate liberal studies' majors were surveyed along with 56 graduate candidates and researchers found some interesting differences. First, the undergraduate students had more experience in mathematics. From demographic data, 65.85% of the liberal studies students claimed to have taken precalculus in high school and only 44. 83% of the graduate students in special education took math classes up through precalculus in high school. The undergraduates used more technology to learn math; for example, 60% said they had used graphing calculators and 20% said they were skilled users. Only 29% of the graduate teacher candidates said they used graphing calculators and none claimed to be skilled users.

Another interesting finding was that the undergraduates claimed to be almost as confident in teaching as the graduate candidates who were usually already teaching. While the undergraduates claimed to have no experience teaching, the graduate candidates claimed to have more; 32.76% claimed to 1-2 years of experience, 32.76% claimed to have 3-5% and 1.72% said they had over 10 years. This was an interest finding given that 46.34% of the undergraduates in the survey responded that they strongly agreed they were confident working as teachers, while only 46.34% of the experienced teachers strongly agreed they felt

Figure 2. Pre and post dynabook learning teaching strategies and structures



confident in their teaching, and 8.95% of the experienced teachers disagreed or strongly disagreed that they felt confident and only 2.44% of the undergraduates disagreed they felt confident. This trend carried on through a number of questions about the students' confidence in teaching children in various areas in math and even when asked about their confidence in generic teaching areas; the undergraduates often felt as confident or more confident than the practicing teachers.

Finally, one last result of interest, when the graduate students were asked what type of preparation they would need before they could teach proportional reasoning effectively, and were given four choices including, 1) nothing could help me teach proportional reasoning, 2) a semester long class, 3) a quick review of the concepts, and, 4) I don't need preparation, I am ready now. Over three fourths (75.86%) said they would need a quick review of the concepts.

Also relevant for this article, in another survey, students in the Spring of 2011 claimed they improved on a number of areas relevant for teaching in special education after using the Dynabook; they felt they knew more about Universal Design for Learning (UDL) and Technological, Pedagogical, Content Knowledge (TPACK), differentiated instruction and strategies for special learners. The chart shows the gains in these areas:

Qualitative Data

The Candidates' Development Process

From the qualitative data the researchers were better able to understand how the students improved and why they scored higher on their LMT and self-efficacy scales after working with the Dynabook. They found that there was a developmental progression in the candidates' ability to learn and to think more deeply about proportional reasoning as the Dynabook tool became more sophisticated. This allowed the professors to more easily design an environment that encouraged more deep and sophisticated conversations from credential candidates. Each time a new version of Dynabook was presented, it was presented to a different group of credential candidates. Each new group started out at about the same level with each assessment that was given. Despite being at the same instructional level before interacting with Dynabook, each progressive class got better (faster and more sophisticated) at discussing problems and different answers as the Dynabook became more sophisticated. The Dynabook had an increasingly greater impact on the professors' ability to move the candidates forward in their thinking about proportional reasoning as the technology developed. Below the researchers included some examples of how this happened.

Prototype

From the discourse analysis, the researchers found that the first time Dynabook was presented to teacher candidates, they wanted to talk about how the tool needed to change. These candidates also talked a lot about how math was usually taught procedurally instead of conceptually. However, during those first sessions, when teacher candidates and faculty talked about the importance of teaching math conceptually, it was not clear everyone was discussing the same concepts. For example, some people were talking about conceptualization as when a child needs to understand why ½ x ¼ = ⅛, --when the multiplier and multiplicand is larger and the answer is smaller. Others defined conceptualization as when children had trouble plugging in numbers to develop equations, or when children had trouble transferring knowledge from one type of problem to another, or when children didn't understand the math language, or when they didn't understand a word problem.

The problem was not that the teacher candidates were talking only about procedures and the faculty was talking about concepts, but that the candidates seemed limited in how they defined "conceptual." The faculty and team members were talking about all the ideas listed above as representing "conceptual teaching and understanding." Teacher candidates often defined conceptual understanding as some type of basic understanding of the problems that resembled what Lobato, Hawley, Druken, and Jacobson (2011) described as "quantitative reasoning." According to those researchers, quantitative reasoning is similar to mathematical structural reasoning in that it goes beyond an understanding of poorly understood algorithms, but it differed in that one's understanding is grounded in one's understanding of measurable quantities in context-rich situations, as opposed to being reliant on the use of mathematical properties. In fact, researchers found it interesting that a lot of our candidates said that understanding math conceptually was easier than remembering the procedures. Here is an example:

Actually, we really liked the brainstorming. We didn't remember the formulas. We knew the math conceptually, but we didn't remember the procedures, so it was hard. (student quote).

This "type of conceptual understanding" made sense given that these candidates had once probably known how to complete these problems and had forgotten both the algorithms and the mathematical properties associated with proportionality, but their experience along with their memory fragments gave them a sense of measurable quantities in context and to them that is what "conceptual understanding" meant.

Version 1

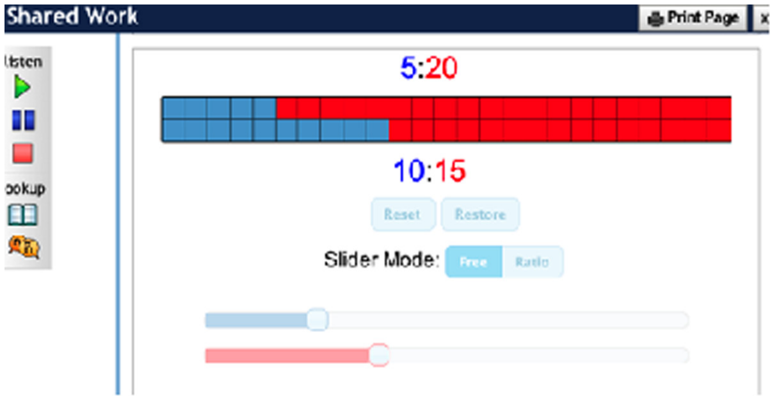
In the first complete version of the Dynabook, the pre-service teacher candidates were being asked to solve ratio problems, then watch the video of children having trouble solving these same ratio problems and then analyze their mistakes. During this iteration, the students talked a lot about using manipulatives in ways that were not quite correct or accurate. Researchers speculated that they were trying to pull from memory what they had learned about math previously. Many were talking about manipulatives in ways that would suggest they were confusing numeracy and proportionality. Below is an example:

Faculty: *Think about it this way. If you have a student who is at the additive level, how do you get him to the multiplicative level?*

Candidate 1: *Taking it from a different subject area (English)… focus on the words and then slowly focus on the computational aspects.*

Candidate 2: *Two glasses of 8 ounces. One in one big bucket and one in a small bucket. One will look like less. They are relating*

Figure 3. Ratio bar visualizer in the dynabook shared space



something different. Has to change their concept of relationships.

Candidate 3: *I don't think she is labeling her stuff. She's not labeling her quantities.*

Faculty: *So labeling her quantities might help her focus on the problem?*

Candidate 3: *I think she's getting confused about minutes and hours. From the table, 7:00 to 8:15. I would not have given her that problem yet because she wasn't ready.*

Faculty: *You said earlier too, that conceptually, she doesn't have the idea yet. She's still looking at the numbers.*

Version 2.0

With the representationally rich version 2.0 of Dynabook, the candidates took a big step forward. This group did not talk about manipulatives because the Dynabook now contained interactive graphic manipulatives. They did not mention teaching math as procedural or conceptual. They seemed to have integrated those two concepts, recognizing that math is about both conceptual understanding and procedural problem solving. They no longer argued that children with disabilities needed to learn procedures before learning concepts. They no longer talked about how conceptual understanding was easier than remembering procedures. And, they no longer defined "conceptual" nebulously as "understanding the overall problem."

Still, students in the class were having a lot of trouble solving the problems. In the math example below, the faculty member is asking the credential candidates to watch a child's video as she struggles with a proportion problem to better understand what she did wrong, and then solve the problem. In the example below the candidates are struggling to analyze the error. The most common mistake made throughout all versions of the Dynabook development was to confuse the part/part and part/whole relationships of proportional problems.

Faculty: *So the salad dressing is 40 parts vinegar and 90 parts olive oil.*

And you have to make 65 ounces.

So, how many ounces of vinegar and how many ounces of olive oil?

Lets see what people did and let's think about this within Khoury's framework:

Level 1) Illogical

Level 2) additive

Level 3) multiplicative

Level 4) ratio

Faculty [reading one response out of many on the Shared Work space]: *Tricia's video. Trisha's understanding of the problem is incomplete*

40/90 can be 4/9 if you divide both by 10.

Thus 65/13 is 5oz per part

4 ounces X 5 ounces + 20 ounces

9 X 5 ounces of vinegar = 45 ounces of Vinegar

Faculty: *Is that right?*

[No one answers]

Faculty: *I want you to think about Khoury's framework, where is this person along that framework? I want you to try to think about it.*
Student: *Ratio right? They are looking at a whole ...*
Faculty: *Well, they understand that ratio is a relationship between two numbers. They definitely understand that. They knew that when they reduced the 40 and 90 and they maintained the ratio. Getting back to Mister Short and Mister tall, because it is correct, it is not additive, so it is only multiplicative or ratio, now we will have a chance to go back and think about it a little bit and we will come back.*

Version 2.1

In the 2.1 version of Dynabook (Fall 2011), the teacher candidates were thinking more deeply about the problems, and coming up with better answers. They seemed more comfortable being open about what they did not know after seeing other people's answers on the shared workspace. Thus, candidates were willing to take risks and to talk about why they made specific decisions when answering specific questions. This meta-cognitive step helped them to understand the problems and their own mathematical thinking. It also helped that the credential candidates could now use a virtual manipulative called the ratio bar visualizer, which was built into this Dynabook iteration. Figure 3 is a picture of the visualizer:

In the class conversation below, the students are discussing reasons why a student might have trouble with one particular problem.

Faculty: *[Describes the red and blue paint problem.] In this problem you need to choose the most likely WRONG answer kids would choose:*
What wrong answer would kids choose?
4 cans red 8 can blue
X cans red 9 cans blue
12, 13, 17, 18
Candidate 1: *Phil chose 17*
Faculty: *Why?*
Candidate 2: *Weird reverse logic*
Candidate 2: *Because Trisha has 8 cans of blue*
Candidate 2: *Because X*2 is 16 and there is only one left and that is close to 17*
Candidate 1: *Is that the correct answer?*
Faculty 2: *The right answer is 13*
Faculty: *How many got it right?*

[About half of the candidates raise their hands]

Faculty: *Why would they choose 13?*
Faculty 2: *What would they do to get the wrong answer?*
Candidate 1: *Put an 8 or a 9….*
Another candidate interrupts: -they would add 9 and 3 is 12.
Faculty: *Where would they get the number 13?*
Candidate 1: *9+4*
Faculty: *They would add 9 and 4 and get 13?*
Candidate 1: *The difference between 4 and 9 is 5 and 8 and 5 is 13.*
Faculty: *Either way they are using the additive reasoning. Not quite where they should be.*

Dynabook 2.2 (Spring 2012)

In the 2.2 version of Dynabook, faculty members presented Dynabook over two class periods for approximately six hours during a curriculum development class focused on advanced math and literacy instruction. The instructor asked the candidates to use some other way to answer the questions in what is called the "mathier" class schedule problem in Dynabook. Students had to decide which school has the mathier schedule. Many of the teacher candidates were quite inventive trying to find technology alternatives for demonstration. Some drew pictures of the ratio, typed explanations and even uploaded and saved different versions of the ratio bar visualizer itself.

That day the instructor had difficulty with the technology, showing the child's video and allowing access to the visualizer so the professor had to explain the child's problem to the class. It was much easier for the teacher candidates to figure out what the child did wrong when they could see the student struggling and refer back to the transcript of the video.

On a positive note, in a large group discussion, when a faculty member was going over candidates' answers on the shared-work space feature of the Dynabook, one candidate said that she was impressed how the variety of explanations in the shared-work space really demonstrated how many different ways a person could solve one problem:

I was just thinking it is really cool how everyone is describing it in a different way, and we can point out to students how there is not just one way to explain this problem, it is a really cool example—look at all these teachers in the room, and they all came up with 20 different ways to explain it.

This was the first time this comment had been made, especially on the first day when the Dynabook was being introduced.

On the second day, there was another technology problem, which was more serious. The Internet browser in the lab was so old the instructor could not access the Dynabook, so the faculty member who was teaching the class was forced to teach the material in the way she had before working with Dynabook, and some students started responding in ways that candidates had at the beginning of the project before using Dynabook. A few started talking about conceptual versus procedural teaching, and in fact one person, who wasn't in the first class when Dynabook was introduced, started arguing that in her experience, special needs children needed to learn procedures before they were ready to learn conceptually. And, a few students started talking about manipulatives in ways that confused ratio with numeracy. It was interesting to have a chance to see the problems that had surfaced at the beginning of the project resurface. This suggested that the changes were not just the result of the instructor becoming more knowledgeable or experienced, but that the technology was helping the college faculty make an impact in her classroom.\

Assessing Mathematical Thinking

One challenge the special education team faced when starting this research project was deciding exactly what they were looking for as they sought to understand how to assess mathematical thinking and how to better understand how to look for improvement as they introduced Dynabook. By exploring these various perspectives, the special education researchers were able to design a rubric for mathematical thinking based on their research outcomes. From the table below, we described the candidate's level of sophistication when thinking about proportionality by examining his or her strengths within four categories, 1) attitudes, 2) mathematics, 3) pedagogy and language, 4) and integration of concepts.

Most of the activities during the Dynabook sessions focused on solving ratio problems and understanding children's mathematical errors. The instructors also requested that candidates develop strategies for helping children learn. For example they were asked to create a virtual dialogue (using Xtranormal and later Dynalog) that required a virtual student and teacher or two virtual students to playact one of the video cases in order to solve a problem. In three different class sessions, the candidates had to write a script to explain how to solve a problem. Below researchers have presented a candidate's comment about that activity at the end of the class discussion:

Coming up with a script was like coming up with a lesson plan, well they are related, but it was different. I am a visual person, so I wanted to throw up a data table, and yet this forced me to think about the actual words I was going to use. Usually I would think of the props and the materials, and assume that I could explain it in the moment and that is not necessarily always

Table 3. Developmental rubrics on teachers' thinking during dynabook project

Attitudes	Less Sophisticated	More Sophisticated	Most Sophisticated
Talking and answering questions in class	Doesn't speak or answer questions even if they think they're correct	May answer some questions if they think they are correct	Are willing to share and explore answers, whether right or wrong
Sharing answers and working in groups	Embarrassed and anxious, afraid of not knowing	Curious but wants to be anonymous	Not afraid to be wrong, confident and curious about other answers
Feelings about knowing math content?	Are confident about teaching math even without content knowledge.	Wants to focus on pedagogy, believes they can learn content when they need it	Understands how content and pedagogy are important and works to integrate
Independence (can work independently or with a colleague without seeking the help of the expert)	Gets stuck on a math problem and needs external help	Gets stuck on a math problem and can experiment with possible solution strategies (but not necessarily get it right)	Never feels stuck on a problem, always believes they can solve on their own eventually or with colleagues
Mathematics	**Less**	**More**	**Most**
Mathematics Understanding	Doesn't understand that Parts/parts/whole	Understands the parts/parts/ whole in some ways	Understands parts /parts / whole relationships in proportionality
Mathematics understanding	Will give answers that are illogical and not see why the answer is illogical	Can see answers are illogical and have no idea how to work on the problem	Can recognize an illogical answer and work to resolve the problem
Ability to solve problems in more than one way	Has trouble solving problem	Can solve problem in one way very well	Can solve problem in more than one way
Pedagogy and Language	**Less**	**More**	**Most**
Language	No mathematics language	Uses some math terms	Uses correct math language often
Ability to explain contents verbally without props	Cannot explain without visuals or props	Can explain with visuals or props, but has trouble without	Can explain well with or without props
Transfer to K-12 classrooms	Doesn't transfer to K-12 classrooms	Transfers if they have no distractions	Transfers even when children are having behavior problems
Knows what to do after analyzing children's problems	Simply **shows** children one procedure for solving a math problem.	**Guides** children through the procedure and allows them to think for themselves, sometimes showing them a couple ways to solve a problem. Sometimes candidate gets confused along the way.	Can **guide** children through multiple ways to solve a problem and help make connections so children understand the problem
Integration of Concepts	**Less**	**More**	**Most**
Understand conceptual versus procedural	Confused about conceptual versus procedural	Not confused, but separates them and has one definition	Integrates the multiple and complex concepts seamlessly
Content and pedagogy	Doesn't understand one or the other – the math or pedagogy	Focuses on one or the other (math or pedagogy)	Integrates math and pedagogy

continued on following page

Table 3. Continued

Attitudes	Less Sophisticated	More Sophisticated	Most Sophisticated
Content and technology	Uses technology for presentation and simple math (calculator)	Uses technology as pedagogy for math. Technology sometimes gets in the way.	Uses technology as integrated part of their pedagogy when teaching math
Analyzing children's problems	Has no where to start when analyzing children's thinking	Starts by simple error analysis or simple developmental categorization	Can analyze children's thinking using multiple theoretical lenses. Can recognize the sophistication of children's thinking based on themes such as integration of important concepts or language.

true. This is forcing us to figure out exactly what we would say to explain it. This is helpful, especially for something complicated.

Although Xtranormal was not part of Dynabook, it does demonstrate how having advanced technology tools available and part of a technology-rich activity allowed this candidate to think about the problem from a different perspective, one in which she was asked to only use language to explain the problem without props. This pushed her to think more deeply about one aspect of math pedagogy, clear explanation. She was thinking about teaching in a different way, using verbal rather than visual cues, relying on language and articulation, rather than on visuals to relay her ideas. This allowed her to expand her thinking (and ours) about the complexity of verbalizing and articulating complex concepts.

In the later iteration of Dynabook where the programmers developed their own more specific type of animation tool, called Dynalog, candidates were able to use a virtual whiteboard that could demonstrate how candidates would use diagrams, manipulatives, objects, and number sentences to teach ratio to struggling learners. This feature enabled faculty to examine and assess candidates' appropriate use of this tools when explaining ratio. Each group of students found different ways to solver various problems and these variations were projected onto this whiteboard and then discussed in class. Universal Design for Learning suggests multiple modes of engagement is essential. For teacher-learners what engaged them most was a video of a struggling K-12 student,their small group error analysis and the development and diverse ways of teaching those preteens how to solve the problem using Dynalog. What we found as second on the list was intense engagement with the *diversity* of responses to that student and problem.

Teachers who had no inkling that a problem could be solved a different way, or that a student could be led on an entirely different path (that limited perspective is the norm, it turns out) came quickly to a deep engagement in those differences, once they could see them side by side, in our "shared posting" implementation. Dynabook doesn't take the place of good pedagogy, it is a tool as part of a classroom activity system that raises the level of candidate thinking and classroom discussion.

CONCLUSION

The researchers involved in this study determined that technology, in this case, the Dynabook, did help SPED faculty teach in ways that pushed teacher candidates to think more deeply about proportional reasoning. From the quantitative data, the resesearchers found that the preservice teachers improved their mathematics and pedagogical content knowledge and they felt more confident. From the qualitative data, they found that each time a new, more sophisticated version of Dynabook came out, the professor in charge of the class was able to elicit more sophisticated conversations from the class. The team also learned that measuring learning and thinking in mathematics is complex and controversial so it was important to think about how to measure student thinking when teaching proportionality and math pedagogy and being able to scan a classroom quickly and make on the spot decisions about whether teachers were learning was important.So the researchers created a rubric that represents a developmental progression that helped them to better define the types of progressions they were seeing from their discussions. Ultimately, the Dynabook not only assisted the students in their learning, it also made the classes more interactive, more interesting and more fun.

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